

Spectral change of σ and ω mesons in a dense nuclear medium

K. Saito

Tohoku College of Pharmacy, Sendai 981-8558, Japan

K. Tsushima and A.W. Thomas

Department of Physics and Mathematical Physics

and

Special Research Centre for the Subatomic Structure of Matter,

University of Adelaide, SA 5005, Australia

Abstract

The spectra of the scalar (σ) and vector (ω) mesons in nuclear matter are studied in detail using the quark-meson coupling (QMC) model. It is shown that above normal nuclear matter density the effects of σ - ω mixing and the decay of the σ into 2π change the spectra of σ and ω mesons considerably. As in Quantum Hadrodynamics we find a remarkable spectral change in the σ meson and the longitudinal mode of the ω meson, namely a two-peak structure.

Keywords: meson spectrum, nuclear matter, mixing effect, quark-meson coupling model

PACS: 21.30.Fe, 21.65.+f, 24.10.Jv, 12.39.Ba, 14.40.-n

Typeset using REVTeX

Recently, medium modifications of hadron properties in a nucleus, which are in some sense precursors of the QCD phase transition, have received a deal of attention [1]. In particular, one of the most interesting phenomena is the spectral change of hadrons in a nuclear medium. At present lattice simulations have mainly been performed for finite temperature (T), with zero chemical potential [2]. Therefore, many authors have investigated hadron properties at finite nuclear densities (ρ_B) using effective theories [1,3–6].

In this paper, using one of these effective theories, namely the quark-meson coupling (QMC) model [4,5], we will study the spectral change of the iso-scalar scalar (σ) and iso-scalar vector (ω) mesons at finite ρ_B (and $T = 0$) in relativistic random phase approximation (RRPA). In Quantum hadrodynamics (QHD) [7] nuclear matter consists of *point-like* nucleons interacting through the exchange of point-like σ and ω mesons. On the other hand, the QMC model could be viewed as an extension of QHD, where the mesons couple to confined quarks (not to point-like nucleons) and the nucleon is described by the MIT bag model. The QMC then yields an effective Lagrangian for a nuclear system [4,5], which has the same form as that in QHD with a *density dependent* coupling constant between the σ and the nucleon (N) – instead of a fixed value. Indeed, from the point of view of the energy of a nuclear system, the key difference between QHD and QMC lies in the σ -N coupling constant, g_s . Although this difference may seem subtle, it leads to many attractive results [4,5]. Therefore, it would clearly be very interesting to investigate the spectra of the mesons in a relativistic framework, including the structural changes of the nucleon in-medium, and compare the QMC results with those given by QHD [8].

First, let us briefly summarize the calculation of the σ and ω meson spectra in symmetric nuclear matter, using QHD [8]. The starting point is the lowest order polarization insertion in the meson propagator. It describes the coupling of the meson, of momentum q , to a particle-hole or nucleon-antinucleon excitation in a nuclear medium. The polarization insertions for the σ , Π_s , and the ω , $\Pi_{\mu\nu}$, are respectively given as

$$\Pi_s(q) = -ig_s^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)G(k+q)] + \frac{3}{2}ig_{\sigma\pi}^2 m_\pi^2 \int \frac{d^4k}{(2\pi)^4} \Delta_\pi(k)\Delta_\pi(k+q), \quad (1)$$

$$\Pi_{\mu\nu}(q) = -ig_v^2 \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)\gamma_\mu G(k+q)\gamma_\nu], \quad (2)$$

where we have added the contribution from the pion-loop to Π_s (the second term on the r.h.s. of eq.(1)) in order to treat the σ more realistically, and g_v and $g_{\sigma\pi}$ are, respectively, the ω -N and σ - 2π coupling constants. The pion propagator, Δ_π , is given by $1/(q_\mu^2 - m_\pi^2 + i\epsilon)$ with the pion mass, m_π (=138 MeV), and $G(k)$ is the self-consistent nucleon propagator (with momentum k) in relativistic Hartree approximation (RHA) given as

$$\begin{aligned} G(k) &= G_F(k) + G_D(k), \\ &= (\gamma^\mu k_\mu^* + M^*) \left[\frac{1}{k_\mu^{*2} - M^{*2} + i\epsilon} + \frac{i\pi}{E_k^*} \delta(k_0^* - E_k^*) \theta(k_F - |\vec{k}|) \right]. \end{aligned} \quad (3)$$

Here $k^{*\mu} = (k^0 - g_v V^0, \vec{k})$ (V^0 is the mean value of the ω field), $E_k^* = \sqrt{\vec{k}^2 + M^{*2}}$ (M^* is the effective nucleon mass in matter) and k_F is the Fermi momentum.

Using the nucleon propagator we can separate the polarization insertion into two pieces: one is the density dependent part, Π^D , which involves at least one power of G_D , and the other is the vacuum polarization insertion, Π^F , which involves only G_F . The former is finite, but the latter is divergent and must be renormalized. We here choose to renormalize such that: for the ω , $\Pi_{\mu\nu}^F$ vanishes at $q_\mu^2 = m_\omega^2$ and $M^* = M$, and for the σ , $\Pi_s^F = \frac{\partial}{\partial q_\mu^2} \Pi_s^F = 0$ at $q_\mu^2 = m_\sigma^2$ and $M^* = M$, where m_ω (783 MeV), m_σ (550 MeV) and M (939 MeV) are respectively the free masses of the ω , σ and the nucleon. The explicit, analytic expressions for not only the density dependent parts, but also the vacuum polarization insertions, are given in Refs. [8–10].

At finite density the scalar and the time component of the vector channels are allowed to mix. This is described by the time component of the mixing polarization insertion:

$$\Pi_m(q) = ig_s g_v \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G(k)\gamma^0 G(k+q)]. \quad (4)$$

In this case there is no vacuum polarization and Π_m vanishes at zero density. (The explicit form can be also found in Ref. [10].)

The RRPA for the meson propagators involves summing of the ring diagrams to all orders. This summation has been discussed by many authors [7–11]. In this paper, using RRPA

we calculate the spectral functions for the mesons, rather than searching for zeros of the inverse of the propagators, because the mesons (especially, the σ meson) have large widths in nuclear matter [12]. The spectral functions of the σ meson, $S_s(q)$, and the transverse (T), $S_T(q)$, and longitudinal (L), $S_L(q)$, modes for the ω meson are respectively given by [8]

$$S_s(q) = -\frac{1}{\pi} \Im m \left[\frac{\Delta_0(1 - d_0\Pi_L)}{\epsilon_{SL}} \right], \quad (5)$$

$$S_T(q) = -\frac{1}{\pi} \Im m \left[\frac{d_0}{1 - d_0\Pi_T} \right], \quad (6)$$

$$S_L(q) = -\frac{1}{\pi} \Im m \left[\frac{d_0(1 - \Delta_0\Pi_s)}{\epsilon_{SL}} \right]. \quad (7)$$

Here $\Pi_T = \Pi_{11}$ or Π_{22} , $\Pi_L = \Pi_{33} - \Pi_{00}$ (we choose the direction of \vec{q} as the z -axis) and ϵ_{SL} is the longitudinal dielectric function

$$\epsilon_{SL} = (1 - d_0\Pi_L)(1 - \Delta_0\Pi_s) - (q_\mu^2/q^2)\Delta_0 d_0\Pi_m^2, \quad (8)$$

with $q = |\vec{q}|$, and the free meson propagators for the σ and ω mesons are

$$\Delta_0(q) = \frac{1}{q_\mu^2 - m_\sigma^2 + i\epsilon} \quad \text{and} \quad d_0(q) = \frac{1}{q_\mu^2 - m_\omega^2 + i\epsilon}. \quad (9)$$

In Ref. [8] the meson masses and the spectral functions in QHD were reported in detail within RRP. A.

Now we are in a position to include the effect of changes in the internal structure of the nucleon in-medium. In order to do so, we consider the following modifications to the approach just obtained for QHD:

(1) meson-nucleon vertex form factor

In QHD the interactions between the mesons and nucleon are *point-like*. However, since both the mesons and nucleon are composite they have finite size. As the simplest example, one could take a monopole form factor [13] at each meson-N vertex:

$$F_N(Q^2) = \frac{1}{1 - q_\mu^2/\Lambda_N^2}, \quad (10)$$

with a cut off parameter $\Lambda_N = 1.5$ GeV. In principle, one could self-consistently calculate the form factor within QMC [14]. However, as such changes are not expected to make a big difference, we use eq.(10) in the following calculation.

(2) density dependence of the coupling constants

In QMC the confined quark in the nucleon couples to the σ field which gives rise to an attractive force. As a result the quark becomes more relativistic in a nuclear medium than in free space. This implies that the small component of the quark wave function, ψ_q , is enhanced in medium [4,5]. The coupling between the σ and nucleon is therefore expected to be reduced at finite density because it is given in terms of the quark scalar charge, $\int_{Bag} dV \bar{\psi}_q \psi_q$. On the other hand, the coupling between the vector meson and nucleon remains constant, because it is related to the baryon number, which is conserved.

Next we consider the widths of the mesons. In eq.(1) we introduced the σ decay into two pions. For the pion-loop, we have added an appropriate counter term to the lagrangian, and prescribe the renormalization in free space. We require the condition [8]: $\Re \Pi_{2\pi}(q_\mu^2) = 0$ at $q_\mu^2 = m_\sigma^2$, where $\Pi_{2\pi}$ is the pion-loop contribution given in eq.(1). Furthermore, we introduce a form factor at the σ - 2π vertex, which is again taken to have a monopole form: $F_\pi(q_\mu^2) = (\Lambda_\pi^2 - m_\sigma^2)/(\Lambda_\pi^2 - q_\mu^2)$ with $\Lambda_\pi = 1.5$ GeV (we choose the same value as Λ_N , for simplicity). Since the σ meson propagator in free space is given in terms of the pion-loop polarization insertion, the free width of the σ meson is

$$\Gamma_\sigma^0 = -\frac{\Im m \Pi_{2\pi}}{m_\sigma} \quad \text{at } q_\mu^2 = m_\sigma^2. \quad (11)$$

Choosing $\Gamma_\sigma^0 = 300$ MeV¹, we find $g_{\sigma\pi} = 18.33$ [8].

It is known that while the ω has a narrow width in free space, this should increase in nuclear matter [16]. Therefore, it is interesting to study the effect of the ω width on the meson spectra. To see this we replace the $i\epsilon$ term in d_0 in eq.(9) by $im_\omega\Gamma_\omega^*$

$$d_0 \rightarrow d_0 = \frac{1}{q_\mu^2 - m_\omega^2 + im_\omega\Gamma_\omega^*}, \quad (12)$$

where

¹Conventionally, the width may be larger than 300 MeV. However, recent theoretical analyses suggest that it is near or below 300 MeV [12,15].

$$\Gamma_{\omega}^* = \Gamma_{\omega}^0 + 40(\text{MeV}) \times (\rho_B/\rho_0), \quad (13)$$

is the in-medium ω width [16] (with the free width, $\Gamma_{\omega}^0 = 9.8$ MeV).

To study the meson spectral functions in nuclear matter, we first have to solve the nuclear ground state within RHA. In QHD the total energy density for nuclear matter is written as [8,9]

$$\mathcal{E} = \mathcal{E}_0 + \frac{1}{2\pi^2} M^2 (M - M^*)^2 \left[\frac{m_{\sigma}^2}{4M^2} + \frac{3}{2} f(1 - 4M^2/m_{\sigma}^2) - 3 \right], \quad (14)$$

where \mathcal{E}_0 has the usual form (in RHA), given in Ref. [7]. Here, $f(z) = 2\sqrt{-z} \tan^{-1} \frac{1}{\sqrt{-z}}$. Note that in Ref. [7] the renormalization condition on the nucleon loops is imposed at $q_{\mu}^2=0$. The second term on the r.h.s. of eq.(14) [8,9] occurs because we chose the renormalization condition for the σ at $q_{\mu}^2 = m_{\sigma}^2$. As measurable quantities cannot depend on this choice, our model gives the same physical quantities as those of Ref. [7].

To take into account the modifications (1) and (2), we replace the σ -N, ω -N and σ -2 π coupling constants in eqs.(1), (2), (4) and (14) by

$$g_s \rightarrow g_s(\rho_B) \times F_N(q_{\mu}^2), \quad g_v \rightarrow g_v \times F_N(q_{\mu}^2) \quad \text{and} \quad g_{\sigma\pi} \rightarrow g_{\sigma\pi} \times F_{\pi}(q_{\mu}^2), \quad (15)$$

where the density dependence of $g_s(\rho_B)$ is given by solving the nuclear matter problem self-consistently in RHA (see Ref. [9]). As in QHD, we have two adjustable parameters in the present calculation: $g_s(0)$ (the σ -N coupling constant at $\rho_B = 0$) and g_v .

Requiring the usual saturation condition for nuclear matter, namely $\mathcal{E}/\rho_B - M = -15.7$ MeV at normal nuclear matter density ($\rho_0 = 0.15 \text{ fm}^{-3}$), we determine the coupling constants $g_s^2(0)$ and g_v^2 : $g_s^2(0) = 61.85$ and $g_v^2 = 62.61$. In the calculation we fix the quark mass to be 5 MeV, while the bag parameters are chosen so as to reproduce the free nucleon mass with the bag radius $R_0 = 0.8$ fm (i.e., $B^{1/4} = 170.0$ MeV and $z = 3.295$ [4,5]). This yields the effective nucleon mass $M^*/M = 0.81$ at ρ_0 and the incompressibility $K = 281$ MeV. (In the present work we do not consider the possibility of medium modification of the meson properties at mean-field level [5].)

Now we present our main results. First, in Fig. 1, we show the density dependence of the coupling constant in QMC. At ρ_0 , g_s decreases by about 9%. The effective nucleon mass is also shown in the figure.

In Fig. 2 we show the transverse spectral function of the ω meson. For low three momentum transfer ($q = 1$ MeV), we show the ω meson spectrum in free space (dotted curve), where the peak reaches about 37 (in units of M^{-2}) at the “invariant” mass $\sqrt{s}(= \sqrt{q_0^2 - |\vec{q}|^2}) = 783$ MeV. We see that the peak first moves downwards but comes back again to the high \sqrt{s} side as the density increases. The width of the T mode increases gradually, because of the in-medium ω width, Γ_ω^* . The spectra for moderate ($q = 750$ MeV) and high ($q = 1.5$ GeV) three momentum transfer are very similar to each other.

We present the longitudinal spectral function of the ω meson in Fig. 3. The spectrum for low q is almost the same as that in S_T , because there is no σ - ω mixing when q vanishes. The difference between S_L and S_T is caused by the σ - ω mixing at finite density. It is very interesting that at moderate q S_L splits into two small, very broad peaks at high density, where the lower peak corresponds to the peak of the σ channel (see Fig. 4). We can see that at high q the higher peak almost disappears and only the lower peak remains at $\rho_B/\rho_0 = 3$.

The spectral function of the σ channel is shown in Fig. 4. The peak of the σ meson first moves downwards and then moves back to the higher \sqrt{s} side — behaviour very similar to that seen for S_T . The width of the σ gradually shrinks at high density. It is remarkable that at moderate or high q the spectrum splits into two peaks at high density because of σ - ω mixing. The higher peak in S_s corresponds to that in S_L , and it is enhanced at high q . This two-peak structure has already been observed in the QHD calculation [8], where the separation into two peaks is seen more sharply than in the present calculation. This is because in QMC the increase of the ω width and the density dependence of the coupling constant moderate the σ - ω mixing in nuclear matter.

To summarize, using QMC we have calculated the spectral functions of the σ and ω mesons in a dense nuclear medium. We have seen that the σ - ω mixing is quite important in dense matter (above $\sim \rho_0$), leading to an interesting spectral change in the σ channel and

the L mode of the ω channel — namely a two-peak structure. Although the width of the σ at finite density is smaller than that in free space, it is, still broad, and the amplitude of the spectrum itself is very small. For the ω the spectrum becomes very broad as the density increases, and the amplitude is sensitive to the width of the ω in matter. We have already reported this remarkable structure in QHD [8], but in the present calculation this is moderated by the in-medium ω width and the decrease of the coupling constant in matter. In future experiments (like dilepton production in heavy ion collisions [1]) it may be possible to obtain information on the spectral change of the mesons. While some experimental possibilities have been proposed for studying meson properties in hot/dense nuclear matter [12,17], in particular, a spectral enhancement of the σ channel near $2m_\pi$ threshold is studied in Ref. [17], it may not be so easy to detect a clear signal of the change in a (cold) dense nuclear medium. Nevertheless, these are important challenge for the future.

This work was supported by the Australian Research Council and the Japan Society for the Promotion of Science.

REFERENCES

- [1] For an overview, see Quark Matter '97, Nucl. Phys. A638 (1998).
- [2] See for example, M. Alford, A. Kapstein, F. Wilczek, hep-lat/9807039; W. Bietenholz, hep-lat/9805014; I.M. Barbour, S.E. Morrison, J.B. Kogut, Nucl. Phys. Proc. Suppl. 63 (1998) 436.
- [3] Chr.V. Christov *et al.*, Nucl. Phys. A510 (1990) 689; G.E. Brown, M. Rho, Phys. Rev. Lett. 66 (1991) 2720; M.K. Banerjee, Phys. Rev. C45 (1992) 1359; M. Asakawa, C.M. Ko, P. Lévai, X.J. Qiu, Phys. Rev. C46 (1992) R1159; E. Naar, M.C. Birse, J. Phys. G19 (1993) 555; M. Herrmann, B.L. Friman, W. Nörenberg, Nucl. Phys. A560 (1993) 411; G. Chanfray, R. Rapp, J. Wambach, Phys. Rev. Lett. 76 (1996) 368; A. Bender, G.I. Poulis, C.D. Roberts, S. Schmidt, A.W. Thomas, Phys. Lett. B431 (1998) 263; K. Tsushima, K. Saito, A.W. Thomas, S.V. Wright, Phys. Lett. B429 (1998) 239; *ibid* (E) B436 (1998) 453; F. Klingl, W. Weise, Acta Phys. Polonica B29 (1998)3225.
- [4] P.A.M. Guichon, Phys. Lett. B200 (1988) 235; P.A.M. Guichon, K. Saito, E. Rodionov, A.W. Thomas, Nucl. Phys. A601 (1996) 349; K. Saito, K. Tsushima, A.W. Thomas, Nucl. Phys. A609 (1996) 339.
- [5] K. Saito, K. Tsushima, A.W. Thomas, Phys. Rev. C55 (1997) 2637; *ibid.* C56 (1997) 566.
- [6] T. Hatsuda, S.H. Lee, Phys. Rev. C46 (1992) R34; X. Jin, D.B. Leinweber, Phys. Rev. C52 (1995) 3344.
- [7] B.D. Serot, J.D. Walecka, Adv. Nucl. Phys. 16 (1986) 1.
- [8] K. Saito, T. Tsushima, A.W. Thomas, A.G. Williams, Phys. Lett. B433 (1998) 243.
- [9] K. Saito, K. Tsushima, A.W. Thomas, nucl-th/9904055, ADP-99-19/T361.
- [10] K. Lim, C.J. Horowitz, Nucl. Phys. A501 (1989) 729.

- [11] S.A. Chin, Ann. of Phys. 108 (1977) 301; K. Saito, T. Maruyama, K. Soutome, Phys. Rev. C40 (1989) 407; T. Hatsuda, H. Shiomi, H. Kuwabara, Prog. Theor. Phys. 95 (1996) 1009.
- [12] S. Chiku, T. Hatsuda, Phys. Rev. D57 (1998) R6; *ibid.* D58 (1998) 076001.
- [13] R. Machleidt, Adv. Nucl. Phys. 19 (1989) 189.
- [14] For example, D.H. Lu, A.W. Thomas, K. Tsushima, A.G. Williams, K. Saito, Phys. Lett. B417 (1998) 217.
- [15] N.A. Törnqvist, M. Roos, Phys. Rev. Lett. 76 (1996) 1575; M. Harada, F. Sannino, J. Schechter, Phys. Rev. D54 (1996) 1991; Phys. Rev. Lett. 78 (1997) 1603; S. Ishida *et al.*, Prog. Theor. Phys. 98 (1997) 1005; T. Hannah, Nordita-1999/27 HE (hep-ph/9905236).
- [16] F. Klingl, N. Kaiser, W. Weise, Nucl. Phys. A624 (1997) 527.
- [17] H.A. Weldon, Phys. Lett. B274 (1992) 133; T. Kunihiro, Prog. Theor. Phys. (Suppl.) 120 (1995) 75; T. Hatsuda, T. Kunihiro, H. Shimizu, Phys. Rev. Lett. 82 (1999) 2840.

FIGURES

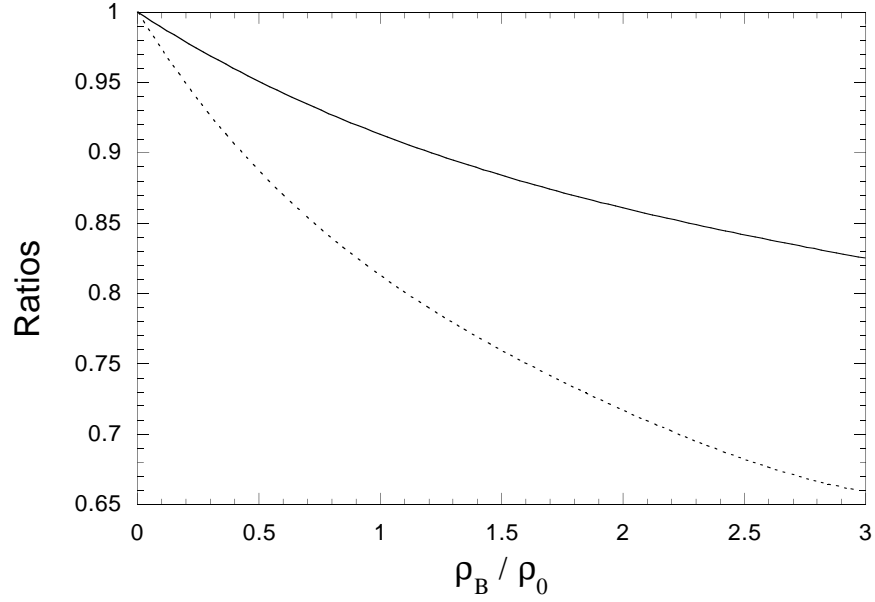


FIG. 1. Density dependence of $g_s(\rho_B)/g_s(0)$ and M^*/M . The solid curve is for the ratio of the coupling constants, while the dotted curve is for the ratio of the nucleon masses.

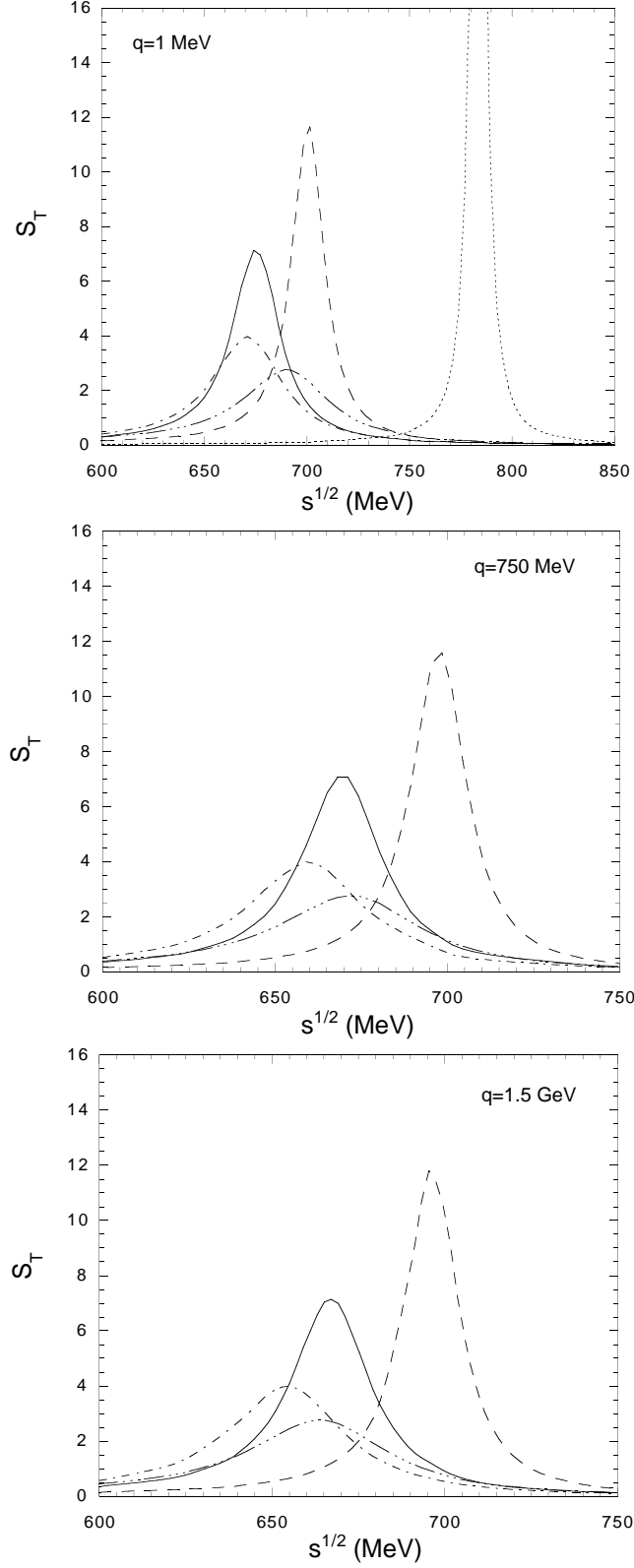


FIG. 2. The transverse spectral function, S_T , for the ω meson (in units of M^{-2}) at three-momentum transfer $q = 1, 750, 1500$ MeV. The dotted curve is for $\rho_B = 0$, while the dashed (solid) [dot-dashed] {3dot-dashed} curve is for $\rho_B/\rho_0 = 0.5$ (1) [2] {3}.

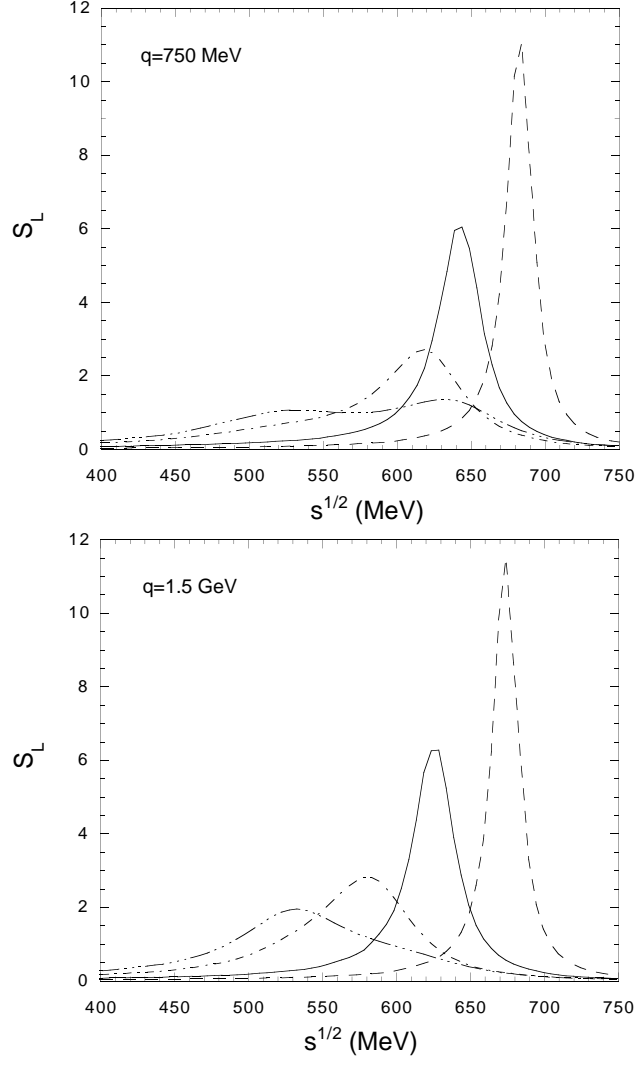


FIG. 3. Same as Fig. 2 but for the longitudinal spectral function, S_L , of the ω meson.

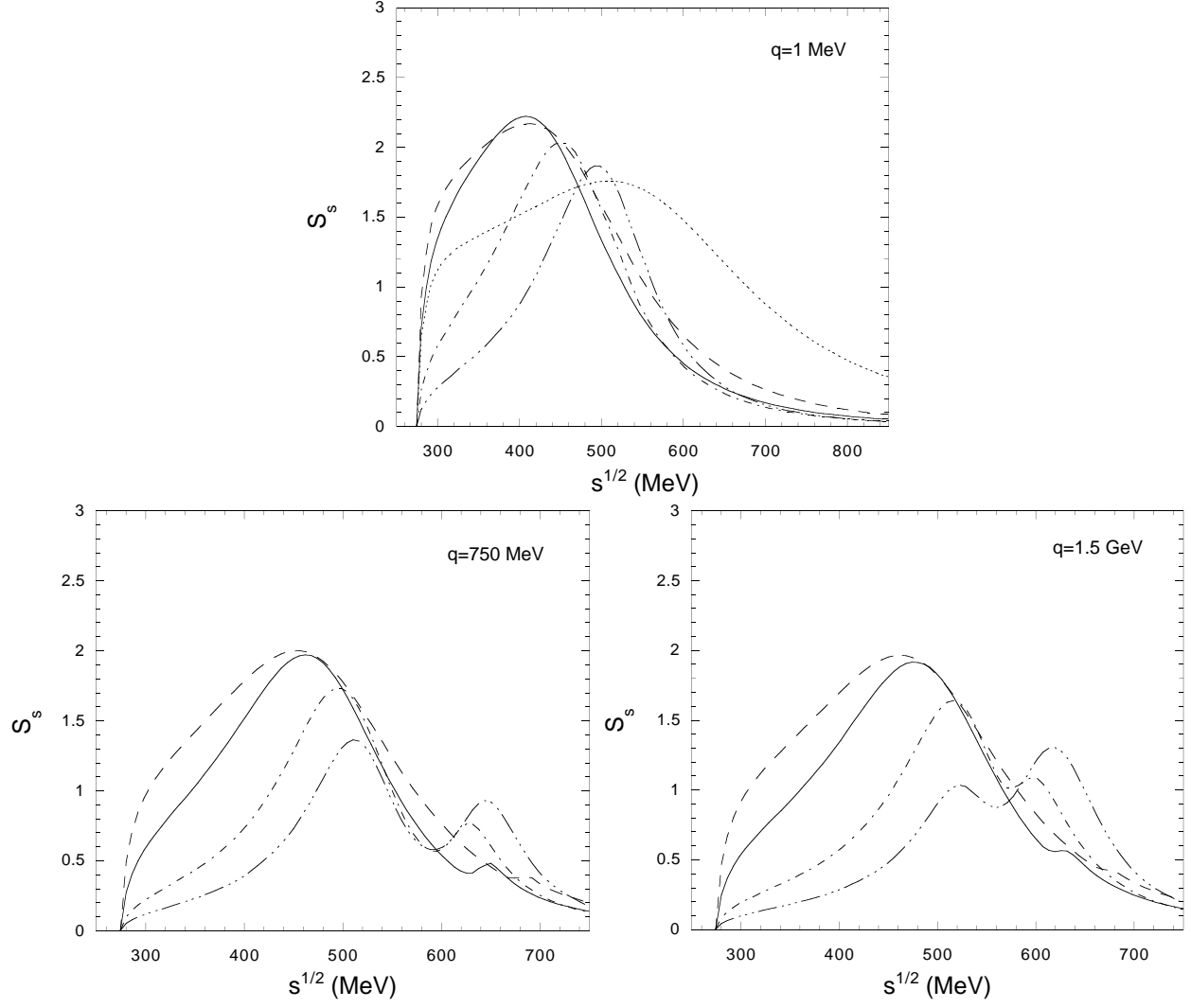


FIG. 4. Same as Fig. 2 but for the spectral function of the σ meson, S_s .